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# Computer vision: models, learning and inference

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*This book is dedicated to Richard Eagle, without whom it would never have been started, and to Lynfa Stroud, without whom it would never have been finished.*

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