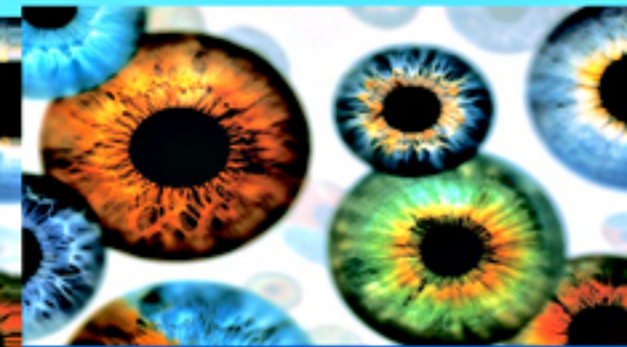


SECOND EDITION

Multiple View Geometry

in computer vision



Richard Hartley and Andrew Zisserman

CAMBRIDGE

CAMBRIDGE

www.cambridge.org/9780521540513

This page intentionally left blank

Multiple View Geometry in Computer Vision
Second Edition

Richard Hartley
Australian National University,
Canberra, Australia

Andrew Zisserman
University of Oxford, UK



CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521540513

© Cambridge University Press 2000, 2003

This publication is in copyright. Subject to statutory exception and to the provision of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published in print format 2004

ISBN-13 978-0-511-18618-9 eBook (EBL)

ISBN-10 0-511-18618-5 eBook (EBL)

ISBN-13 978-0-521-54051-3 paperback

ISBN-10 0-521-54051-8 paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Dedication

This book is dedicated to Joe Mundy whose vision and constant search for new ideas led us into this field.

Contents

<i>Foreword</i>	<i>page</i>	<i>xi</i>
<i>Preface</i>		<i>xiii</i>
1 Introduction – a Tour of Multiple View Geometry		1
1.1 Introduction – the ubiquitous projective geometry		1
1.2 Camera projections		6
1.3 Reconstruction from more than one view		10
1.4 Three-view geometry		12
1.5 Four view geometry and n -view reconstruction		13
1.6 Transfer		14
1.7 Euclidean reconstruction		16
1.8 Auto-calibration		17
1.9 The reward I : 3D graphical models		18
1.10 The reward II: video augmentation		19
PART 0: The Background: Projective Geometry, Transformations and Esti- mation		23
<i>Outline</i>		<i>24</i>
2 Projective Geometry and Transformations of 2D		25
2.1 Planar geometry		25
2.2 The 2D projective plane		26
2.3 Projective transformations		32
2.4 A hierarchy of transformations		37
2.5 The projective geometry of 1D		44
2.6 Topology of the projective plane		46
2.7 Recovery of affine and metric properties from images		47
2.8 More properties of conics		58
2.9 Fixed points and lines		61
2.10 Closure		62
3 Projective Geometry and Transformations of 3D		65
3.1 Points and projective transformations		65
3.2 Representing and transforming planes, lines and quadrics		66

3.3	Twisted cubics	75
3.4	The hierarchy of transformations	77
3.5	The plane at infinity	79
3.6	The absolute conic	81
3.7	The absolute dual quadric	83
3.8	Closure	85
4	Estimation – 2D Projective Transformations	87
4.1	The Direct Linear Transformation (DLT) algorithm	88
4.2	Different cost functions	93
4.3	Statistical cost functions and Maximum Likelihood estimation	102
4.4	Transformation invariance and normalization	104
4.5	Iterative minimization methods	110
4.6	Experimental comparison of the algorithms	115
4.7	Robust estimation	116
4.8	Automatic computation of a homography	123
4.9	Closure	127
5	Algorithm Evaluation and Error Analysis	132
5.1	Bounds on performance	132
5.2	Covariance of the estimated transformation	138
5.3	Monte Carlo estimation of covariance	149
5.4	Closure	150
PART I: Camera Geometry and Single View Geometry		151
	<i>Outline</i>	152
6	Camera Models	153
6.1	Finite cameras	153
6.2	The projective camera	158
6.3	Cameras at infinity	166
6.4	Other camera models	174
6.5	Closure	176
7	Computation of the Camera Matrix P	178
7.1	Basic equations	178
7.2	Geometric error	180
7.3	Restricted camera estimation	184
7.4	Radial distortion	189
7.5	Closure	193
8	More Single View Geometry	195
8.1	Action of a projective camera on planes, lines, and conics	195
8.2	Images of smooth surfaces	200
8.3	Action of a projective camera on quadrics	201
8.4	The importance of the camera centre	202
8.5	Camera calibration and the image of the absolute conic	208

8.6	Vanishing points and vanishing lines	213
8.7	Affine 3D measurements and reconstruction	220
8.8	Determining camera calibration K from a single view	223
8.9	Single view reconstruction	229
8.10	The calibrating conic	231
8.11	Closure	233
PART II: Two-View Geometry		237
	<i>Outline</i>	238
9	Epipolar Geometry and the Fundamental Matrix	239
9.1	Epipolar geometry	239
9.2	The fundamental matrix F	241
9.3	Fundamental matrices arising from special motions	247
9.4	Geometric representation of the fundamental matrix	250
9.5	Retrieving the camera matrices	253
9.6	The essential matrix	257
9.7	Closure	259
10	3D Reconstruction of Cameras and Structure	262
10.1	Outline of reconstruction method	262
10.2	Reconstruction ambiguity	264
10.3	The projective reconstruction theorem	266
10.4	Stratified reconstruction	267
10.5	Direct reconstruction – using ground truth	275
10.6	Closure	276
11	Computation of the Fundamental Matrix F	279
11.1	Basic equations	279
11.2	The normalized 8-point algorithm	281
11.3	The algebraic minimization algorithm	282
11.4	Geometric distance	284
11.5	Experimental evaluation of the algorithms	288
11.6	Automatic computation of F	290
11.7	Special cases of F -computation	293
11.8	Correspondence of other entities	294
11.9	Degeneracies	295
11.10	A geometric interpretation of F -computation	297
11.11	The envelope of epipolar lines	298
11.12	Image rectification	302
11.13	Closure	308
12	Structure Computation	310
12.1	Problem statement	310
12.2	Linear triangulation methods	312
12.3	Geometric error cost function	313
12.4	Sampson approximation (first-order geometric correction)	314

12.5	An optimal solution	315
12.6	Probability distribution of the estimated 3D point	321
12.7	Line reconstruction	321
12.8	Closure	323
13	Scene planes and homographies	325
13.1	Homographies given the plane and vice versa	326
13.2	Plane induced homographies given F and image correspondences	329
13.3	Computing F given the homography induced by a plane	334
13.4	The infinite homography H_∞	338
13.5	Closure	340
14	Affine Epipolar Geometry	344
14.1	Affine epipolar geometry	344
14.2	The affine fundamental matrix	345
14.3	Estimating F_A from image point correspondences	347
14.4	Triangulation	353
14.5	Affine reconstruction	353
14.6	Necker reversal and the bas-relief ambiguity	355
14.7	Computing the motion	357
14.8	Closure	360
PART III:	Three-View Geometry	363
	<i>Outline</i>	364
15	The Trifocal Tensor	365
15.1	The geometric basis for the trifocal tensor	365
15.2	The trifocal tensor and tensor notation	376
15.3	Transfer	379
15.4	The fundamental matrices for three views	383
15.5	Closure	387
16	Computation of the Trifocal Tensor \mathcal{T}	391
16.1	Basic equations	391
16.2	The normalized linear algorithm	393
16.3	The algebraic minimization algorithm	395
16.4	Geometric distance	396
16.5	Experimental evaluation of the algorithms	399
16.6	Automatic computation of \mathcal{T}	400
16.7	Special cases of \mathcal{T} -computation	404
16.8	Closure	406
PART IV:	N-View Geometry	409
	<i>Outline</i>	410
17	N-Linearities and Multiple View Tensors	411
17.1	Bilinear relations	411
17.2	Trilinear relations	414